JOURNAL OF MATERIALS SCIENCE: MATERIALS IN MEDICINE **11** (2000) 261–265

Applications of an anisotropic parameter to cortical bone

S. S. KOHLES
Departments of Biomedical and Mechanical Engineering, Worcester Polytechnic Institute, Worcester, MA 01609-2280, USA
Department of Clinical Science, Tufts University School of Veterinary Medicine, Grafton, MA, USA
Departments of Physiology and Orthopedics, University of Massachusetts Medical School, Worcester, MA, USA
E-mail: kohles@wpi.edu

An equational description of the extent of the anisotropy in cortical bone is presented from both the perspective of plane stress (two–dimensional stress state) and plane strain (three–dimensional stress state). The orthotropic elastic properties that are incorporated in these states are used to provide a more thorough and refined description of planar and volumetric anisotropy in comparison to the commonly used ratio of elastic moduli. The resulting anisotropic parametric equations (η^{σ} and η^{ε}) are applied to the elastic material properties measured from cortical bone within rats, dogs, cows and humans as reported in 12 previous studies. The resulting calculated parameters reduce the typically nine independent properties down to three parameters which in turn represent the degree of anisotropy within the three orthogonal planes of symmetry as are common in cortical bone. It was found that no statistical difference existed between the plane stress versus plane strain parameter in all but two studies ($p > 0.10$). Planar and volumetric anisotropies were compared to the isotropic condition ($\eta^{\sigma} = \eta^{\varepsilon} = 1.0$) for all of the included studies. All of the studies reported cortical bone properties that were volumetrically anisotropic ($p < 0.05$), however, a common plane of isotropy was noted in the radial-circumferential (1–2) plane ($p > 0.05$). Future use of these parametric equations will allow further illucidation of the issue of mesomechanical and micromechanical levels of anisotropy within other tissues and materials of interest.

1. Introduction

The evaluation of the elastic properties of cortical bone is a pursuit that has ranged from the simple to complex in terms of the quantity and quality of the measured properties. In early investigations of the mechanical properties of bone, simple structural tests were completed in order to quantify basic stiffness parameters [1]. As biomechanical tests incorporated more thorough experimental and theoretical mechanics techniques, material properties of bone were soon measured including Young's and shear moduli [2, 3]. Generally, cortical bone was viewed as an isotropic material where the assumptions for simple structural tests satisfied the need for extensive elastic evaluation.

As more high-resolution techniques were applied to bone as well as a hightened appreciation of its microstructure, an increase in the number of measured properties were reported. Through the application of composite theories, full orthotropic descriptions revealed direction dependencies in its elasticity [4–6]. Techniques such as ultrasonic elasticity and scanning electron microscopy revealed detailed material parameters [7, 8]. This increase in elastic property measurement allowed for more normalized comparisons of bone and a thorough description of the responses to physiologic perturbations.

The basis for the understanding of the anisotropic nature of bone is drawn from a number of descriptive levels in the engineering heirarchy representing its mechanical state. This heirarchy extends from the structural level (organ and limb) to the mesomechanical and micromechanical levels (tissue) down to molecular mechanics (cells). As revealed in studies of bone elastic properties, this tissue is both anisotropic and heterogenous, thus the noted properties are dependent upon the orientation and location at which they are evaluated. The local mesomechanical (tissue level) properties are generally described as orthotropic with reference to the longitudinal, radial, and circumferential axes of long bones. This orthotropic characteristic is typical of both primary and secondary cortical bone. In primary bone, concentric layers of mineral (lamellae) comprise the periosteal to endosteal thickness (plexiform bone). Along the boundary of each lamella are small cavities (lacunae) which contain a single bone cell (osteocyte). Radiating from each lacuna are tiny canals (canaliculi) into which the osteocytes extend their cytoplasmic processes. In the secondary bone that is established in higher order mammals during bone removal (due to osteoclasts) and replacement (due to osteoblasts), the basic structural unit is the longitudinally oriented osteon, which consists of concentric lamellar rings surrounding a Haversian canal. Each osteon is bounded by a cement line and the Haversian canals are connected by transversely oriented Volkemann's canals. The Haversian and non-Haversian system constructions, with their composite arrays of longitudinal and transverse canals, bias the mechanical response towards the directionally dependent, orthotropic nature of cortical bone. At the micromechanical level, a further contribution to orthotropy is revealed in the longitudinal arrangement of collagen fibers with transversely connected crosslinks, all of which are supported by a matrix of mineral, parallelopiped shaped crystals. Thus the directional dependence of bony tissue is reiterated at multiple levels within its constitutive heirarchy.

With the increase in the number of measured and comparable parameters arises a truer quantification of the degree of tissue anisotropy. In the present paper, an equational method is proposed regarding the elastic description of tissue on three fundamental levels. First, this description will reduce the complex array of elastic properties down to a single elastic parameter as a description for each material plane. Second, the resulting parameter can account for the influence of all relevant elastic properties including any possible shear and longitudinal couplings. Thirdly, the equations are applied to three-dimensional elastic data from previous biological studies thus creating a unique set of comparisons between species, primary and secondary bone, quadripeds and bipeds, and measurement techniques.

2. Theory development

Normalized elastic properties are, by definition, independent of tissue geometry. However, the physical dispersion of the tissue is critical in the deformation response to a given loading arrangement. If the evaluated sample has a thickness that is much less than the other transverse dimensions it is assumed to exist in a state of plane stress (stress in the thickness direction is zero). An example of this state would be long bone cortex thickness compared with long bone circumference and diaphyseal length. Surface tissue may also be in a state of plane stress as no contact force would exist to create a stress in the third dimension. Conversely, if a tissue sample has equal relative dimensions or represents a location of multiaxial loading, i.e., a musculotendonous connection at a bony tubercle or a tissue sample that is internal and far from a surface, then it may be presumed to be in a state of plane strain (an off axis dimension is constrained or has small levels of strain compared to the loaded plane). Thus an equational description of the extent of the planar anisotropy can be developed from both the perspective of plane stress (two-dimensional stress state) and plane strain (three-dimensional stress state) via the elastic properties that are involved in these states [9]. The present paper applies such a development.

3. Planar anisotropy parameters

The development of the planar anisotropy parameters presented here is expanded for the application to composite biomaterials such as cortical bone. This effort addresses the identification of varying types of elastic symmetry [10] and the characterization of levels of anisotropy [11, 12].

3.1. Plane stress

In the absence of body forces, plane stress problems in the theory of anistropic elasticity [13, 14] reduce to a determination of the local stress function $F(x, y)$ which in turn satisfy the fourth order partial differential equation

$$a_{22}\frac{\partial^4 F}{\partial x^4} - 2a_{26}\frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66})\frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16}\frac{\partial^4 F}{\partial x \partial y^3} + a_{11}\frac{\partial^4 F}{\partial y^4} = 0 \quad (1)$$

where $a_{ij}(i, j = 1 \text{ to } 6)$ are the compliance coefficients of the generalized Hooke's law (often seen as s_{ij}). The stiffness matrix is then defined as $c_{ij} = a_{ij}{}^{-1}$. The compliance coefficients in terms of the recognized engineering constants are

$$\begin{aligned} a_{11} &= 1/E_{11} & a_{12} &= -\nu_{21}/E_{22} & a_{13} &= -\nu_{31}/E_{33} \\ a_{22} &= 1/E_{22} & a_{23} &= -\nu_{32}/E_{33} & a_{33} &= 1/E_{33} \\ a_{44} &= 1/G_{23} & a_{55} &= 1/G_{13} & a_{66} &= 1/G_{12} \end{aligned} \quad (2)$$

where E_{ii} denote Young's moduli, ν_{ij} Poisson ratios, and G_{ij} shear moduli (for i, $j = 1$, 2, and 3 denoting the material axes such as the radial, circumferential, and longitudinal orientations, respectively, for long bones). For orthotropic elasticity in the 1–2 plane, the characteristic equation of Equation 1 is

$$a_{11}\mu^4 + (2a_{12} + a_{66})\mu^2 + a_{22} = 0 \quad (3)$$

where the real and complex roots μ_k are

$$-\mu_1\mu_2 = \sqrt{\frac{a_{22}}{a_{11}}} = \sqrt{\frac{E_{11}}{E_{22}}} \quad (4)$$

and

$$-i(\mu_1 + \mu_2) = \sqrt{\frac{2(a_{22} + a_{12}) + a_{66}}{a_{11}}} = \sqrt{2\left(\frac{E_{11}}{E_{22}} - \nu_{12}\right) + \frac{E_{11}}{G_{12}}} \quad (5)$$

where $i = \sqrt{-1}$. We then define the 1–2 plane stress anisotropic parameter as

$$\eta_{12}^{\sigma} = \frac{1}{2}\sqrt{2\left(\frac{E_{11}}{E_{22}} - \nu_{12}\right) + \frac{E_{11}}{G_{12}}} \quad (6)$$

where only planar elastic properties (four independent constants) are incorporated to characterize the planar (two-dimensional) stress environment. From Equation 6, the 2–3 and 1–3 plane stress anisotropic parameters are then

$$\eta_{23}^{\sigma} = \frac{1}{2}\sqrt{2\left(\frac{E_{22}}{E_{33}} - \nu_{23}\right) + \frac{E_{22}}{G_{23}}} \quad (7)$$

and

$$\eta_{13}^{\sigma} = \frac{1}{2}\sqrt{2\left(\frac{E_{11}}{E_{33}} - \nu_{13}\right) + \frac{E_{11}}{G_{13}}} \quad (8)$$

respectively. Complimentary parameters of η_{21}^{σ}, η_{32}^{σ} and η_{31}^{σ} can also be determined from the above equations. Thus, the degree of anisotropy of an orthotropic material can now be evaluated by three parameters. Equations 6 through 8 are then necessary, but not sufficient, to catagorize a material as isotropic. For an isotropic material, $\eta_{ij}{}^{\sigma} = 1$, $E_{ii} = E_{jj}$ and $G_{ij} = E_{ii}/[2(1 + \nu_{ij})]$ as measured from the i–j plane.

3.2. Plane strain

In a similar manner of development, the plane strain problem is reduced to the determination of the local stress function $F(x, y)$ which again satisfy the fourth order partial differential equation similar to Equation 1 and is given by

$$\beta_{22}\frac{\partial^4 F}{\partial x^4} - 2\beta_{26}\frac{\partial^4 F}{\partial x^3 \partial y} + (2\beta_{12} + \beta_{66})\frac{\partial^4 F}{\partial x^2 \partial y^3} - 2\beta_{16}\frac{\partial^4 F}{\partial x \partial y^3} + \beta_{11}\frac{\partial^4 F}{\partial y^4} = 0 \quad (9)$$

where β_{ij} are the reduced coefficients of deformation and are related a_{ij} by the following formula:

$$\beta_{ij} = a_{ij} - \frac{a_{ik}a_{jk}}{a_{kk}} \text{ for } \left\{ \begin{array}{l} k = 1 \text{ and } i,j = 2,3,4 \\ k = 2 \text{ and } i,j = 1,3,5 \\ k = 3 \text{ and } i,j = 1,2,6 \end{array} \right\} \quad (10)$$

For orthotropic elasticity in the 1–2 plane, the characteristic equation of Equation 9 is

$$\beta_{11}\mu^4 + (2\beta_{12} + \beta_{66})\mu^4 + \beta_{22} = 0 \quad (11)$$

where the real and complex roots μ_k are

$$-\mu_1\mu_2 = \sqrt{\frac{\beta_{22}}{\beta_{11}}} = \sqrt{\frac{E_{11}(1 - \nu_{32}\nu_{23})}{E_{22}(1 - \nu_{31}\nu_{13})}} \quad (12)$$

and

$$-i(\mu_1 + \mu_2) = \sqrt{\frac{2(\beta_{22} + \beta_{12}) + \beta_{66}}{\beta_{11}}} = \sqrt{\frac{E_{11}E_{33}}{E_{33} - E_{11}\nu_{31}^2}\left[\frac{2(1 - \nu_{21})}{E_{22}} - \frac{2\nu_{32}(\nu_{31} + \nu_{32})}{E_{33}} + \frac{1}{G_{12}}\right]} \quad (13)$$

Thus the 1–2 plane strain anisotropic parameter is defined as

$$\eta_{12}^{\varepsilon} = \frac{1}{2}\sqrt{\frac{E_{11}E_{33}}{E_{33} - E_{11}\nu_{31}^2}\left[\frac{2(1 - \nu_{21})}{E_{22}} - \frac{2\nu_{32}(\nu_{31} + \nu_{32})}{E_{33}} + \frac{1}{G_{12}}\right]} \quad (14)$$

where the out-of-plane elastic properties are now incorporated to characterize the three–dimensional stress environment which maintains the planar strain field. The 2–3 and 1–3 plane strain anisotropic parameters are then

$$\eta_{23}^{\varepsilon} = \frac{1}{2}\sqrt{\frac{E_{22}E_{11}}{E_{11} - E_{22}\nu_{12}^2}\left[\frac{2(1 - \nu_{32})}{E_{33}} - \frac{2\nu_{13}(\nu_{12} + \nu_{13})}{E_{11}} + \frac{1}{G_{23}}\right]} \quad (15)$$

and

$$\eta_{13}^{\varepsilon} = \frac{1}{2}\sqrt{\frac{E_{11}E_{22}}{E_{22} - E_{11}\nu_{21}^2}\left[\frac{2(1 - \nu_{13})}{E_{33}} - \frac{2\nu_{23}(\nu_{21} + \nu_{23})}{E_{22}} + \frac{1}{G_{13}}\right]} \quad (16)$$

respectively. Complimentary parameters of η_{21}^{ε}, η_{32}^{ε} and η_{31}^{ε} can also be determined. Again, Equations 14 through 16 are then necessary, but not sufficient, to catagorize a material as isotropic. For an isotropic material, $\eta_{ij}{}^{\varepsilon} = 1$, $E_{ii} = E_{jj}$ and $G_{ij} = E_{ii}/[2(1 + \nu_{ij})]$ as measured from the i–j plane.

4. Cortical bone application and analysis

Nine independent material properties (Young's and shear moduli and Poisson's ratios) for cortical bone have been reported for four different species representing 19 different treatment or control groups (Table I). The array of samples represents both primary and secondary cortical bone analyzed using either mechanical or ultrasonic elasticity techniques. The developed planar anisotropic parameters presented here were applied to each of the three orthogonal planes typifying the orthotropic elastic arrangement of cortical bone (Table II).

Paired t–tests were used to analyze the resulting parameters in order to establish any statistical difference between plane stress versus plane strain derived descriptions. Absolute values of the differences of all parameters from the isotropic condition ($\eta = 1.0$) were tested using one-sample t–tests (hypothesized mean ≤ 0) to address volumetric anisotropy. Planar anisotropy was evaluated as a comparison of η_{ij} and η_{ji} to the isotropic condition for the ij plane.

5. Results and discussion

An equational application has been proposed which offers another anisotropic description of tissue. This description reduces the complex array of elastic properties down to a single comparable elastic parameter for each material plane. The resulting planar parameter accounts for the influence of all of the relevant elastic properties [9] including any possible shear and longitudinal couplings that may exist in plane stress and plane strain scenarios. With the advent of this development, the equations have been applied to three-dimensional elastic data from previous biological studies.

The development of the two anisotropic parameters is based on similar approaches to solving the stress function $F(x, y)$. Due to the nature of increased complexity from a two-dimensional to a three-dimensional stress state, the plane strain parameter, η^{ε}, is a more thorough formulation. However, in paired comparisons within each study, only the elastic properties determined by Lang [4] ($p = 0.078$) and Burris [15] ($p = 0.033$)

TABLE I Summary of nine independent material properties including Young's moduli (E_{ij} in GPa), shear moduli (G_{ij} in GPa) and Poisson's ratios (ν_{ij}) for cortical bone, as reported for four different species representing 19 different treatment or control groups. The subscripts $i, j = 1, 2,$ and 3 denote the material axes as the radial, circumferential, and longitudinal orientations, respectively, for long bones

Species	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Cow	Human	Cow	Dog	Human	Human	Human	Cow	Cow
Trtmnt	Dwrf	DwrfGH	Cont	2G	Cont	GH	Cont	GH	Cont	GH	Cont	Cont	Cont	Cont	Cont	Cont	Cont	Cont	Cont
Bone	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Phalanx	Femur	Femur	Femur	Femur	Femur	Tibia	Femur	Femur
Age	51days	51days	74days	74days	9mo	9mo	20mo	20mo	31mo	31mo									
Ref.	20	20	21	21	19	19	19	19	19	19	4	5	6	7	7	22	17	18	15
Property																			
E11	11.68	9.48	13.39	13.28	15.32	16.45	16.87	17.34	19.20	19.39	11.3	18.8	11.6	12.8	12.0	11.5	6.91	6.97	10.79
E22	14.91	9.17	14.30	16.30	20.29	19.74	19.76	21.93	23.94	22.33	11.3	18.8	11.6	15.6	13.4	11.5	8.51	6.97	12.24
E33	17.98	13.84	19.13	17.84	22.13	24.28	23.94	24.60	25.75	24.29	22.0	27.4	21.9	20.1	20.0	17.0	18.4	20.9	18.9
G23	3.65	3.37	7.03	6.72	8.21	7.78	8.16	8.04	8.46	8.45	5.4	8.71	6.99	6.67	6.23	3.3	4.91	6.9	5.96
G31	4.27	3.50	5.86	5.96	6.96	7.10	6.98	7.21	7.42	6.98	5.4	8.71	6.26	5.68	5.61	3.3	3.56	6.9	4.47
G12	4.37	3.31	4.97	5.76	6.59	6.30	6.27	6.54	7.06	7.25	3.8	7.71	5.29	4.68	4.53	3.6	2.41	2.2	3.38
v31	0.373	0.260	0.430	0.507	0.537	0.440	0.433	0.449	0.403	0.399	0.396	0.281	0.206	0.454	0.371	0.46	0.32	0.44	0.42
v21	0.314	0.484	0.350	0.229	0.299	0.329	0.348	0.314	0.347	0.327	0.484	0.310	0.38	0.366	0.422	0.58	0.62	0.55	0.51
v32	0.268	0.308	0.417	0.406	0.376	0.353	0.351	0.329	0.298	0.326	0.390	0.281	0.307	0.341	0.35	0.46	0.31	0.44	0.33
v13	0.247	0.184	0.311	0.394	0.374	0.303	0.308	0.313	0.301	0.334	0.203	0.193	0.109	0.289	0.222	0.31	0.12	0.15	0.24
v12	0.257	0.522	0.338	0.196	0.224	0.298	0.323	0.274	0.297	0.293	0.484	0.312	0.302	0.282	0.376	0.58	0.49	0.55	0.45
v23	0.227	0.200	0.308	0.388	0.373	0.292	0.297	0.296	0.280	0.320	0.203	0.193	0.205	0.265	0.235	0.31	0.14	0.15	0.22

Note: Treatment (Trtmnt) and control (Cont) groups include dwarf rats with growth hormone supplementation (DwrfGH) and without (Dwrf), hypergravity treated controls (2G), and growth hormone treatments during aging (GH).

demonstrated a strong statistical difference between parameters ($\eta^{\sigma} < \eta^{\varepsilon}$). In future applications it is recommended to use the simpler plane stress anisotropic parameter as a measure of the extent or degree of material anisotropy.

In a comparison between complimentary anisotropy parameters, η_{ij} and η_{ji}, it is apparent that typically if $\eta_{ij} < 1.0$ then $\eta_{ji} > 1.0$. This arrangement is due to the dominance (greater stiffness) of the orientation symbolized by the first of the two subscripts (when > 1.0). This result may offer further description to the elastic nature of tissues.

With regard to the evaluation of the elastic data from the included studies, statistical analyzes of the anisotropic nature was undertaken. Overall, all sets of properties demonstrated a global volumetric anisotropy ($p < 0.05$). However, statistical isotropy ($p > 0.05$) was noted in all but two groups in the 1–3 plane, in all but five groups in the 2–3 plane, and in all but one group in the 1–2 plane based on one-sample t-tests of the complimentary (η_{ij} and η_{ji}) plane stress and plane strain parameters (Table II). Often these results reflect the initial planar isotropies that were assumed within the specific study. An evaluation of the symmetrical elastic nature would need to be undertaken on individual specimens so that group averages can be calculated and compared. Additionally, the parameters may reduce the number of statistical comparisons between experimental groups within a study. Previously, up to 10 separate stastical tests were needed in order to conclude upon a tissue's level of isotropy [16]. Earlier work applied spatial averages of stiffness and compliance coefficients to determine levels of volumetric anisotropy [11, 12]. The resulting parameters kept axial and shear elastic properties separate during the evaluation of anisotropies. However, those efforts and the parameters presented in

TABLE II The resulting anisotropic parameters as developed from plane stress and plane strain assumptions and calculated for each of the orthotropic elastic descriptions. The statistically anisotropic notations are the result of comparisons of η_{ij} and η_{ji} to the isotropic condition (1.0) for the i–j plane

Species	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Rat	Cow	Human	Cow	Dog	Human	Human	Human	Cow	Cow
Trtmnt	Dwrf	DwrfGH	Cont	2G	Cont	GH	Cont	GH	Cont	GH	Cont	Cont	Cont	Cont	Cont	Cont	Cont	Cont	Cont
Bone	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Femur	Phalanx	Femur	Femur	Femur	Femur	Femur	Tibia	Femur	Femur
Age	51days	51days	74days	74days	9mo	9mo	20mo	20mo	31mo	31mo									
Ref	20	20	21[a]	21[a]	19	19[b]	19	19	19	19	4	5	6[b]	7[b]	7[b,c]	22	17	18	15[b]
Plane stress																			
η31	1.22	1.21	1.09	1.04	1.06	1.11	1.11	1.11	1.11	1.11	1.23	1.12	1.21	1.13	1.16	1.29	1.40	1.18	1.23
η21	1.12	0.97	1.03	1.07	1.09	1.08	1.07	1.11	1.12	1.07	1.00	0.98	0.93	1.10	1.03	1.00	1.06	1.01	1.09
η32	1.28	1.22	1.02	0.99	1.00	1.08	1.05	1.06	1.06	1.04	1.23	1.12	1.15	1.07	1.11	1.29	1.23	1.1	1.12
η13	0.98	1.00	0.91	0.89	0.88	0.92	0.93	0.93	0.93	0.99	0.88	0.92	0.88	0.90	0.90	1.06	0.86	0.68	0.93
η12	0.99	0.98	0.99	0.96	0.95	0.98	0.99	0.99	0.98	0.99	1.00	0.98	0.95	1.00	0.97	1.00	0.96	1.01	1.02
η23	1.17	0.99	0.89	0.94	0.95	0.97	0.95	0.95	1.00	0.99	0.88	0.93	0.82	0.94	0.91	1.03	0.84	0.68	0.90
Plane strain																			
η31	1.28	1.19	1.15	1.14	1.16	1.18	1.16	1.17	1.15	1.15	1.26	1.15	1.28	1.19	1.20	1.28	1.43	1.25	1.24
η21	1.14	0.97	1.04	1.08	1.10	1.09	1.08	1.12	1.12	1.08	1.00	0.98	0.96	1.11	1.04	1.00	1.10	1.01	1.10
η32	1.33	1.20	1.07	1.08	1.07	1.12	1.08	1.10	1.08	1.06	1.26	1.15	1.20	1.12	1.14	1.28	1.24	1.25	1.12
η13	0.95	1.07	0.87	0.82	0.81	0.87	0.90	0.89	0.94	0.96	0.89	0.90	0.85	0.87	0.88	1.17	0.91	0.65	0.95
η12	0.98	0.98	0.99	0.96	0.95	0.98	0.99	0.99	0.99	1.00	1.00	0.98	0.91	0.99	0.96	1.00	0.93	1.01	1.02
η23	1.17	1.07	0.84	0.88	0.90	0.94	0.93	0.98	1.02	0.97	0.89	0.90	0.80	0.90	0.90	1.17	0.88	0.65	0.91

[a] Anisotropic in 1–3 plane ($p < 0.05$)
[b] Anisotropic in 2–3 plane ($p < 0.05$)
[c] Anisotropic in 1–2 plane ($p < 0.05$)

this paper provide some consistency in results as evident in the analysis of similar data such as those provided by Knets and Malmeisters [17]. In general, this work contributes to the categorization process of the elastic isotropic versus anisotropic performance of cortical bone.

Through quantification of the complex three-dimensional elastic properties, there still exists a need for conclusion of the issue of planar isotropy, especially as more and more characteristics are incorporated into tissue adaptation models and orthopedic component designs. The anisotropic measuring parameters in this paper should help to simplify this issue in terms of a representation of all interacting elastic properties. Future research will investigate further the elastic arrangement of other connective tissues as well as the musculoskeletal reality of plane stress and plane strain environments.

Acknowledgments

The author acknowledges insightful and revealing conversations with Dr Andrew Rapoff, valuable advice from Dr Ray Vanderby Jr, and the Whitaker Foundation's financial contribution to the field of biomedical engineering.

References

1. F. G. EVANS, Stress strain in bones: their relation to fractures and osteogenesis (C. C. Thomas, Springfield, IL, 1957).
2. D. T. REILLY and A. H. BURSTEIN, *J. Bone Jt Surg.* **56A** (1974) 1001.
3. D. R. CARTER and D. M. SPENGLER, *Clin. Orthop* **135** (1978) 192.
4. S. B. LANG, *IEEE Trans., Biomedical Engineering* **17** (1970) 101.
5. H. S. YOON and J. L. KATZ, *J. Biomechanics* **9** (1976) 459.
6. W. C. VAN BUSKIRK, S. C. COWIN and R. N. WARD, *J. Biomechanical Engineering* **103** (1981) 67.
7. R. B. ASHMAN, S. C. COWIN, W. C. VAN BUSKIRK and J. C. RICE, *J. Biomechanics* **17** (1984) 349.
8. S. F. LIPSON and J. L. KATZ, *ibid.* **17** (1984) 231.
9. A. J. RAPOFF, Private communication.
10. S. C. COWIN and M. M. MEHRABADI, *J. Biomechanics* **22** (1989) 503.
11. J. L. KATZ and A. MEUNIER, *ibid.* **20** (1987) 1063.
12. *Idem., J. Mater. Sci.: Mat. Med.* **1** (1990) 1.
13. S. G. LEKHNITSKII Anisotropic plates (translated from 2nd Russian ed. by S. W. Tsai and T. Cheron, New York, Gordon and Breach Science Publishers Inc., 1968) pp. 24–29, 171.
14. A. P. BORESI and K. P. CHONG Elasticity in engineering mechanics, (Elsevier Science Publisher New York, 1987).
15. C. L. BURRIS, PhD Dissertation, (Tulane University, New Orleane 1983).
16. R. B. ASHMAN, G. ROSINIA, S. C. COWIN, M. G. FONRENOT and J. C. RICE, *J. Biomechanics* **18** (1985) 717.
17. I. KNETS and A. MALMEISTERS in ''Mechanics of Biological Solids: Proceedings of Euromech Colloquium 68'', edited by G. Branks Varna, Bulgaria (Bulgarian Academy of Sciences, Sofia, 1977) p. 133.
18. J. D. AMBARDAR and C. D. FERRIS, *Biomedical Sci. Instr.* **12** (1976) 23.
19. S. S. KOHLES, G. D. CARTEE and R. VANDERBY JR, *J. Med. Eng. & Tech.* **20** (1996) 157.
20. S. S. KOHLES, D. A. MARTINEZ, J. R. BOWERS, R. VANDERBY JR and A. C. VAILAS, *Annals of Biomedical Eng.* **25** (1997) 77.
21. S. S. KOHLES, J. R. BOWERS, A. C. VAILAS and R. VANDERBY JR, *Calcified Tissue International* **59** (1996) 214.
22. D. T. REILLY and A. H. BURSTEIN, *J. Biomechanics* **8** (1975) 393.

Received 5 November 1997
and accepted 20 April 1999